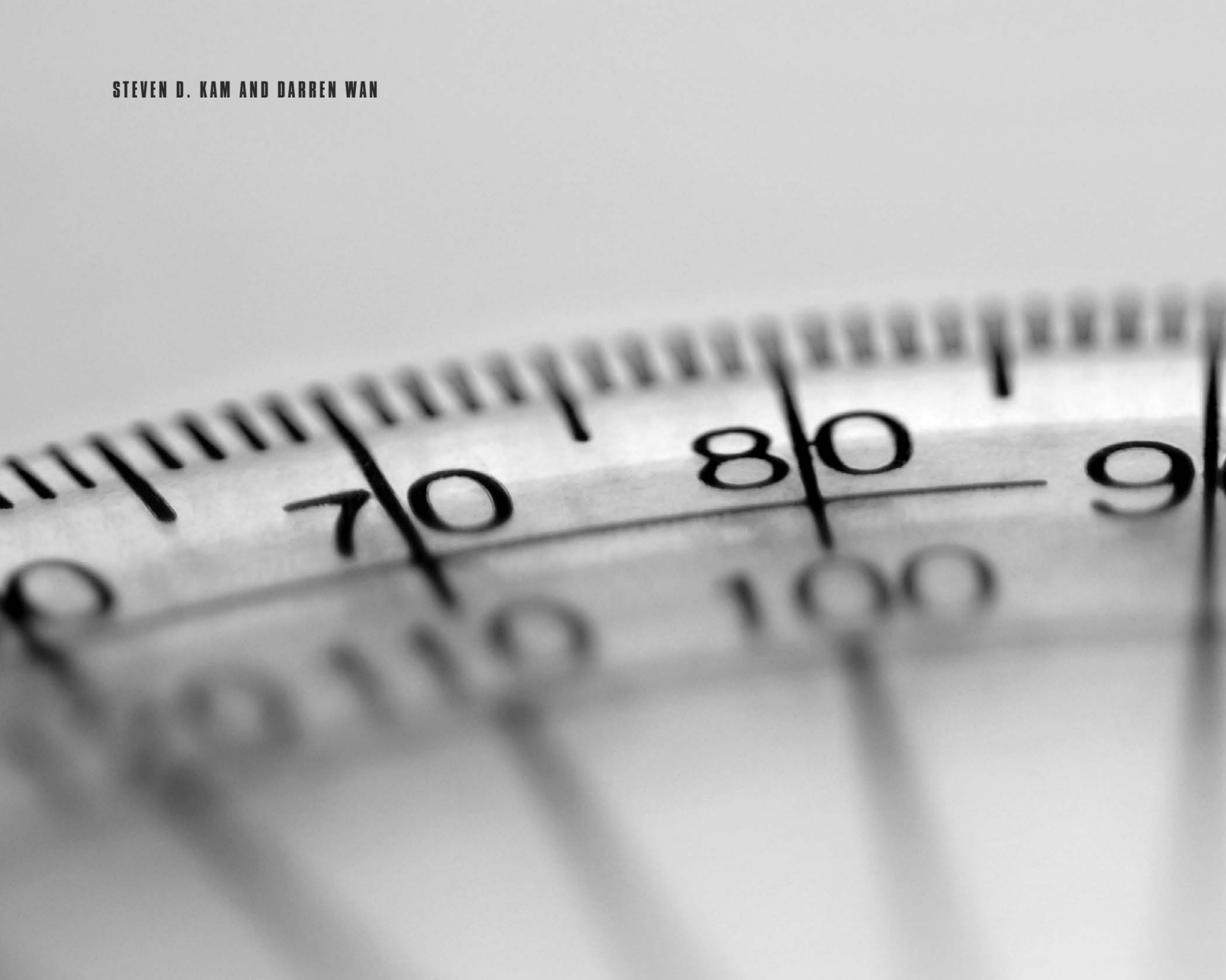
Valuation Strategies

NOVEMBER/DECEMBER 2010

Measuring The Incremental Discount For Lack of Marketability

Steven D. Kam and Darren Wan



Recent developments in determining the discount for lack of marketability rely on quantifiable parameters of a specific asset rather than empirical data on historical transactions.

MEASURING THE INCREMENTAL DISCOUNT FOR LACK OF MARKETABILITY

The current thinking

regarding the adjustment or discount for the lack of marketability (DLOM) has been based, in part, on empirical studies of private placements involving securities restricted under Rule 144 (known as "letter stock") that compare the stock's private placement price to its freely-traded price. The FMV Restricted Stock Study ("FMV Study") and Valuation Advisors' Pre-IPO Study ("Pre-IPO Study") are examples of these empirical studies. More recent insight, however, has led to the development of approaches that do not rely on empirical data from historical transactions. Instead, these approaches focus on quantifiable parameters that are specific to the asset being valued. One such approach applies the Black-Scholes-Merton (BSM) option pricing methodology to value protective put options.

Use of Protective Put Options to Quantify Lack of Marketability

In a protective put option calculation, the cost of purchasing a European put option to protect against downward price changes in the underlying asset is considered a proxy for the cost of marketability. For private entity valuation purposes, the cost of a put option is the price an investor would pay, on the valuation date, for the right to sell the underlying asset at a guaranteed price at the maturity of the option (thus assuring its marketability). The investor's net position[1] as of the valuation date is the value of the underlying asset minus the price of the put option. From this calculation, the DLOM can be observed as the price of the put option divided by the marketable value of the underlying asset.

For example, assume an investor owns a security with a marketable value of $100 and is restricted from selling the security for five years. The investor purchases a five-year put option for $35, providing her the right (but not the obligation) to sell the asset for $100 at the end of five years. By paying $35, or 35% of the marketable security price, the investor has assured herself liquidity[2] at the end of the option term and has eliminated all downward pricing risk. The investor's overall investment position at the time of the option purchase is reduced by 35% from $100 to $65, but the entire risk attendant to identifying a buyer to purchase the security has been neutralized.

As a proxy for DLOM, the rationale in this example assumes that an investor who has the choice between purchasing two identical securities—except that one is completely marketable and the other is completely nonmarketable—would pay 35% less for the nonmarketable security than she would for the marketable security. The investor would pay 35% less for the nonmarketable security because he would pay 35% of the security's price to purchase a protective put option to assure liquidity and downside price protection at the end of her investment holding period.

Using the Black-Scholes-Merton Model

One way to calculate the cost of a protective put option is through the BSM model (see Exhibit 1). This is comprised of six inputs:

1. Security price.
2. Strike price.
3. Volatility.
4. Term.
5. Risk-free rate.
6. Dividend yield.

In a protective put option, the strike price is equal to the value of the underlying asset. In a business valuation context, setting the strike price to equal the underlying asset price gives the investor the right, at the maturity of the put option, to sell the asset at the price determined at the applicable valuation date. The volatility of the underlying asset can be measured by calculating the implied volatilities of traded options on that asset or volatilities calibrated from traded options on comparable assets. Alternatively, the volatility can be measured by calculating the asset's historical trading volatility, or, if no trading data exists, by calculating the historical trading volatility of comparable assets. The term of the protective put option is the holding period for the security. Last, the risk-free rate corresponds with the term of the put option. Exhibit 2 compares the relationship of the inputs to the BSM model to the price of the protective put option and the corresponding DLOM.

Application to Nonmarketable Securities

Protective put option methodology is particularly useful for investors with assets that are restricted from sale, whether by legal restrictions or market conditions. For example, assume that a valuation has been performed on a nonmarketable security (Security A) using the comparative analysis of publicly traded companies. The value of Security A was determined to be $100 at the marketable, minority level of value. To account for the differences in

STEVEN D. KAM, ASA, is a managing director and DARREN WAN, CFA, is a vice-president at Cogent Valuation, San Francisco, California.

1 "Position" is an investor's stake in a particular security or market. A long position equals the number of shares owned; a short position equals the number of shares owed by a dealer or an individual. Dictionary of Finance and Investment Terms, 4th ed. (Barron's Educational Series, 1995).

2 Discussion of the differences between marketability and liquidity is outside of the scope of this article. The terms will be used interchangeably without distinction for the purposes of this article.

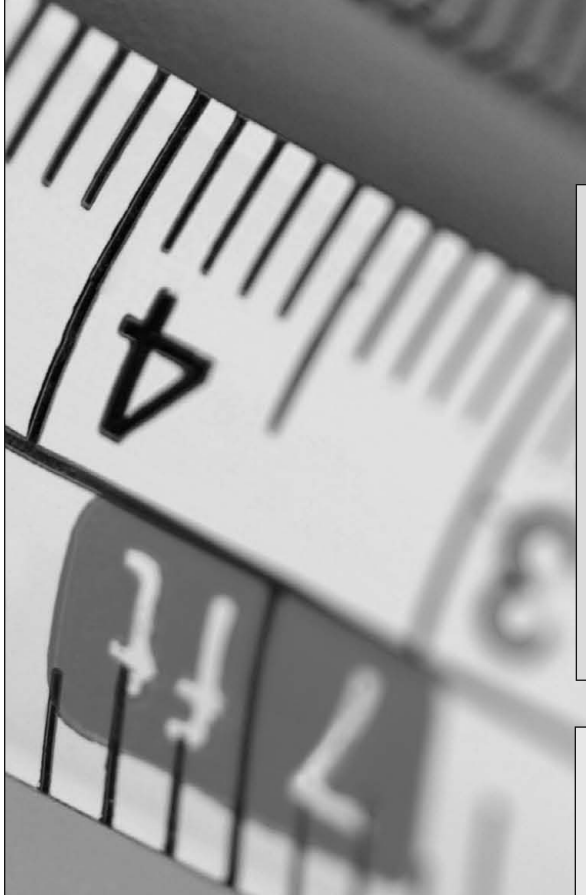


EXHIBIT 1
Black-Scholes-Merton Model

$p = c - S + Xe^{-rT}$
$c = Se^{-qT}N(d1) - Ke^{-rT}N(d2)$
$d1 = \ln(S/K) + (r - q + \sigma^2/2)T / \sigma\sqrt{T}$
$d2 = d1 - \sigma\sqrt{T}$

Where:

p	=	price of a written put
c	=	price of a written call
S	=	value of the underlying asset at the applicable valuation date
K	=	exercise price or strike price
σ	=	annualized volatility of the underlying stock
T	=	time to expiration (in years)
r	=	continuously compounded risk-free rate
q	=	continuously compounded dividend yield
N	=	cumulative probability function for a standardized normal distribution
e	=	a mathematical constant; the base of the natural logarithm

EXHIBIT 2
Effect of Inputs to the BSM Model on Put Option Prices and DLOM

			Put Option Value	DLOM
Price of the underlying security[a]	(1) S	△	No Effect	No Effect
		▽	No Effect	No Effect
Exercise price[a]	(2) K	△	No Effect	No Effect
		▽	No Effect	No Effect
Annualized volatility of the underlying stock	(3) σ	△	Increases	Increases
		▽	Decreases	Decreases
Time to expiration (in years)	(4) T	△	Increases	Increases
		▽	Decreases	Decreases
Continuously compounded risk-free rate	(5) r	△	Decreases	Decreases
		▽	Increases	Increases
Continuously compounded dividend yield	(6) q	△	Increases	Increases
		▽	Decreases	Decreases

a The asset and exercise price are set equal to each other because the asset price is the value the investor desires to protect. Since S and X maintain a constant 1:1 relationship, changes to S and X do not affect the relationship between the price of the underlying security and the put option value.

EXHIBIT 3
Lack of Marketability Calculation for an LP Interest

Price of the underlying asset at the valuation date /a/	$100.00
Strike price	$100.00
Annualized volatility of the underlying asset /b/	0.45
Option term (in years)	5.00
Continuously compounded risk-free rate /c/	4.00%
Continuously compounded dividend yield	0.00%
Value of the Protective Put Option	$26.60
Put Option Divided by Underlying Asset Price	26.60%
Marketable, Minority Value of LP Interest	$100.00
Less: Cost of Marketability ($100.00 * 26.6%)	($26.60)
Non-Marketable, Minority Value of LP Interest	**$73.40**

/a/ Marketable, minority value of the LP or LLC interest, not net asset value.

/b/ Volatility matched to option term.

/c/ Risk-free rate to maturity is matched to option term.

Security A's marketability relative to the marketable securities to which it was compared, a complete DLOM must be taken on Security A to develop a nonmarketable, minority indication. In Exhibit 3, a protective put option is calculated for Security A, the underlying asset. An investor in Security A has no public or recognized market to sell the asset and faces restrictions on sale or transfer. A term of five years represents the holding period for the investment.

For a security with a five-year holding period, the investor can purchase a five-year put option for 26.6% of the underlying asset price, thereby reducing his or her overall net position by 26.6% but eliminating illiquidity risk and downward pricing risk. By implementing this investment strategy, it is assumed that the investor would pay up to 73.4% of the marketable price, or $73.40, for the nonmarketable LP interest. Thus, the investor would pay 26.6% less for the nonmarketable security, because it would cost 26.6% of the marketable price to purchase a protective put option to assure liquidity and downside price protection at the end of the five-year holding period.

Effect of Option Term on Option Value. In Exhibit 4, the values of two protective put options are calculated on the same underlying asset to demonstrate the effect of the term on the value of the protective put option, assuming that all other inputs are held constant. The first protective put option has a term of six months; the second has a term of five years. The put option with the six-month term is purchased so that after six months the owner of the underlying asset is assured liquidity for the asset at the stated strike price. Similarly, the owner of the five-year put option is assured liquidity at the stated strike price for the underlying asset after five years has passed. Risk-free rates and volatilities corresponding to the terms of the put options are also presented in Exhibit 4.

Clearly, the difference between option terms has a material effect on the values of the protective puts (assuming that all other inputs are held constant). As shown in Exhibit 2, the five-year put option costs more than the six-month option because the investor is protected over a longer period of time.

Application to LP or LLC Interests

Protective put option methodology can also be used to determine DLOMs for limited partner interests ("LP interests") and nonmanaging, member interests ("LLC interests") in closely held entities that own real estate, securities, or a combination of both. First, the marketable, minority value of the LP or LLC interest is determined, typically using comparable publicly traded interests. Then, depending on the degree of marketability of the comparable publicly traded interests, an appropriate DLOM is applied to the LP or LLC interest.

LP and LLC interests share several common attributes with real estate limited partnership (RELP) interests traded in the secondary market.[3] Over time, the pricing of RELP interests in the secondary market has been considered an appropriate proxy for the pricing of LP and LLC interests. There are two methods to value LP and LLC interests: the net asset value method and the price-to-yield method.

Net Asset Method. The net asset value method derives an indication of value from the application of a price-to-NAV multiple ("NAV Multiple") as determined from a secondary market analysis of RELP interests, with corroborating data from other sources, including traders, investors, and asset managers.

The Price-to-Yield Method. The price-to-yield method considers market-based yields ("Required Yield") of traded partnerships as a source of pricing data for LP and LLC interests. The price-to-yield method capitalizes expected distributions by dividing the market-required yield into the representative distributions. Market pricing information for the traded limited partnerships is deemed to be a useful proxy for LP and LLC interests, since LP, LLC, and RELP interests have many attributes in common, including:

1. Rights and restrictions provided in their governing documents.
2. Underlying assets (primarily real estate properties).
3. Investment opportunities (income-producing investments).
4. Operating performance.

RELP interests that are traded in the secondary market exhibit some marketability, albeit partial marketability. For example, unlike a security listed on the NYSE, a RELP trades periodically (not daily), privately, and in small, irregular volumes. Further, unlike a security listed on the NYSE, the purchase or sale of an LP or LLC interest is a customized transaction between parties without the benefits of an established marketplace. Therefore, RELP interests are neither completely marketable (because they do not trade in meaningful volume over public exchanges such as the NYSE), nor completely nonmarketable (because a secondary market exists for their resale). Here, trades are made with far less frequency and with a greater level of uncertainty as to participant interest and pricing than demonstrated over public exchanges. No identifiable market exists for closely held LP or LLC interests.

Investors' pricing of RELP interests in the secondary market includes their partial marketability. In contrast, LP or LLC interests have no marketability, because they are interests in closely held entities that do not trade in any secondary market. Therefore, any valuation of closely held LP or LLC interests based on RELPs must be adjusted to compensate investors for the complete lack of marketability of these interests relative to the partial marketability of the traded RELP interests.

Application to Securities with Partial Marketability

When developing a DLOM for valuation indications that reflect partial marketability, the DLOM derived from methods such as the FMV Study or the Pre-IPO Study must be reduced. DLOMs derived from the FMV Study and the Pre-IPO Study represent complete lack of marketability discounts that are used to adjust from the (completely) marketable level of value to the (completely) nonmarketable level of value. The multiples derived from RELPs already include partial lack of marketability discounting. Protective put option methodology can be used to quantify the difference between

[3] Kam, Schroeder, and Smith, "The Market Pricing of Syndicated LPs and the Valuation of FLPs," Trusts & Estates 35 (February 1996).

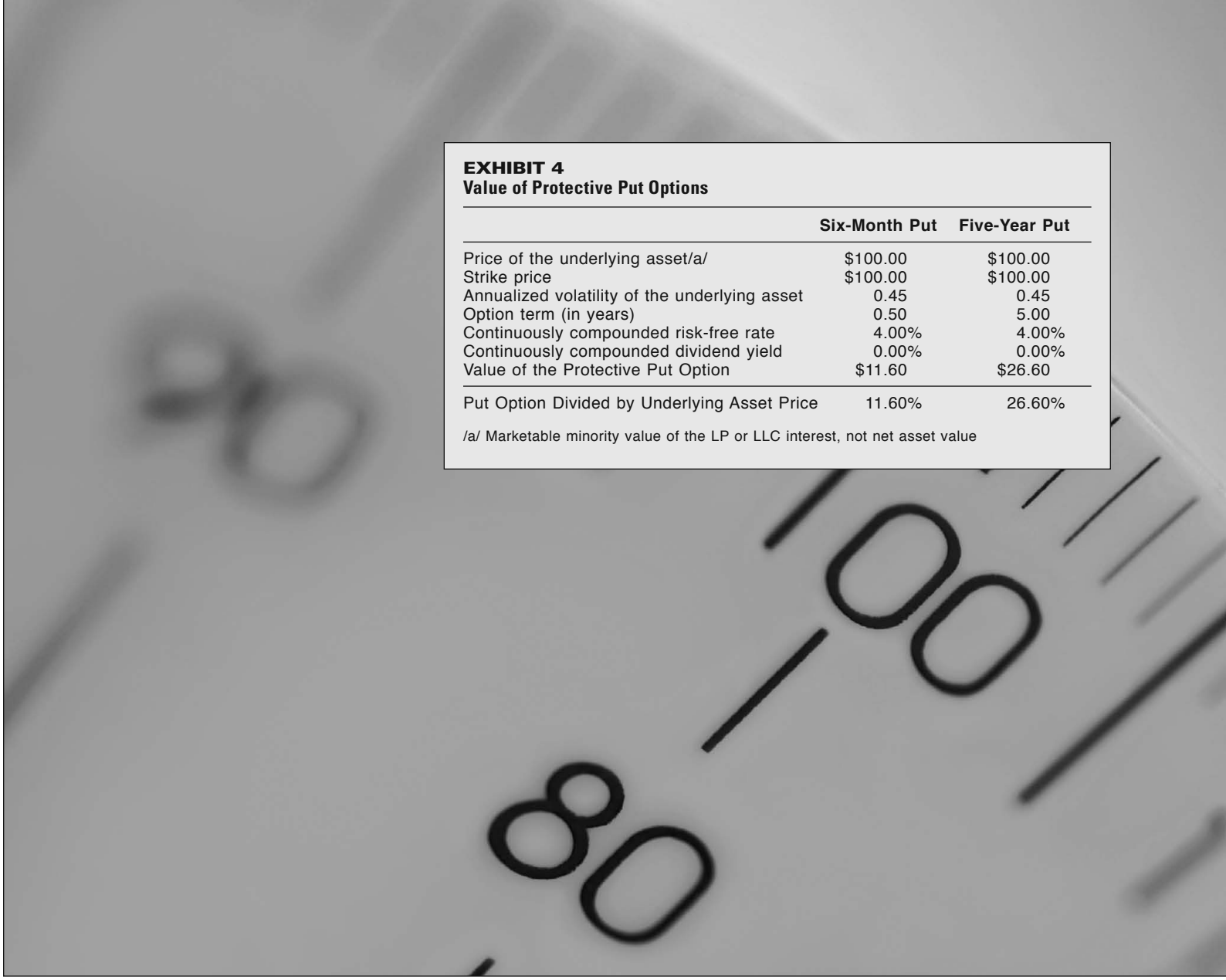


EXHIBIT 4
Value of Protective Put Options

	Six-Month Put	Five-Year Put
Price of the underlying asset/a/	$100.00	$100.00
Strike price	$100.00	$100.00
Annualized volatility of the underlying asset	0.45	0.45
Option term (in years)	0.50	5.00
Continuously compounded risk-free rate	4.00%	4.00%
Continuously compounded dividend yield	0.00%	0.00%
Value of the Protective Put Option	$11.60	$26.60
Put Option Divided by Underlying Asset Price	11.60%	26.60%

/a/ Marketable minority value of the LP or LLC interest, not net asset value

complete lack of marketability and partial lack of marketability. This methodology can be useful for adjusting DLOMs that reflect partial marketability when the situation requires incrementally more DLOM for a security that should have a full DLOM built into its price.

As discussed, an investor who wants to assure liquidity and price protection could purchase a protective put option. Moreover, the length of time required to effectuate the sale of an asset corresponds to the price an investor will pay to assure liquidity at a future date. With a completely marketable security, the investment can be quickly liquidated for cash. For example, an investor holding shares of Johnson and Johnson, Inc. stock could call her broker and receive cash within three days. An investor holding shares in a thinly traded partnership or other security with partial marketability seeking cash would not have the certainty of a readily available market or even an estimate of the timing of the sale. Consequently, there is greater uncertainty as to the timing of the sales transaction and sales price. The expected timeframe to achieve liquidity of an asset with partial marketability will vary according to the specific characteristics of each asset and the depth of market activity.

Assume an investor owns a LP interest with a marketable value of $100 determined through a RELP comparison analysis. Comparing the underlying business of the LP interest to companies within the FMV Study and Pre IPO Study yields a DLOM of 30%, which reflects the difference between complete marketability and complete nonmarketability. As discussed previously, however, the indications of value developed from RELPs already include a partial lack of marketability discount. Thus, applying the full lack of marketability discount of 30% from the FMV and Pre-IPO Studies would overestimate the appropriate discount to the LP interest. Instead of applying the complete 30% discount, it would be more appropriate to compute a discount that quantifies the incremental lack of marketability between a partially marketable RELP interest and the completely nonmarketable private LP interest.

To calculate the incremental discount, the values of two protective put options on the LP interest are calculated. An investor in the closely held LP has no public or established market to sell his assets and faces restrictions on the sale or transfer contained in the LP governing documents. A term of

five years is assumed to represent a reasonable holding period. To reflect this inherent difference between a completely marketable security and a security with partial marketability, a proportion is developed that quantifies the difference between an asset with partial marketability and a completely nonmarketable asset.

In Exhibit 4, the protective put option used to quantify partial marketability is given a term of six months. The protective put option with a term of five years is used to quantify complete lack of marketability. The put option with the six-month term provides assurance to the owner of the underlying asset that in six months he will receive the stated strike price for his asset if exercised. Similar logic applies to the five-year option.

Unlike the example in Exhibit 4, which illustrated the effect on option value of varying the term of the option (other inputs held constant), the example in Exhibit 5 calculates a real-world example using varied inputs—variables that change with the term of the option.

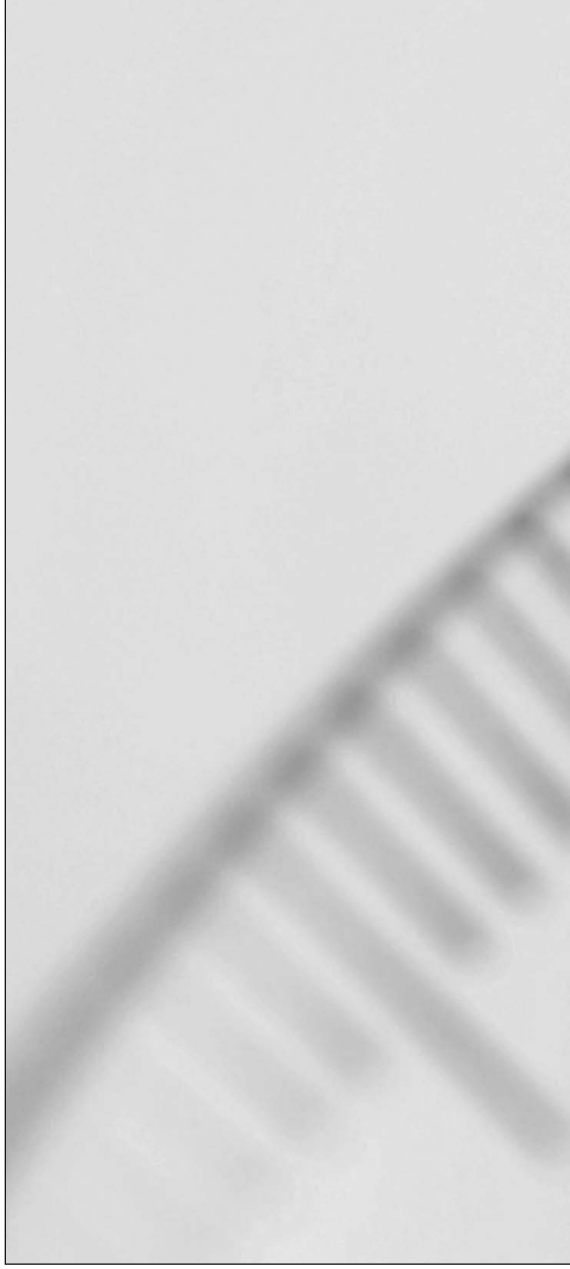
The implied lack of marketability discounts was computed as follows:

1. First, the put option values ($16.20 and $26.60) were divided by the marketable, minority value of the LP or LLC interest ($100.00). These proportions (16.2% and 26.6%) represents the cost to assure liquidity and price protection for six months and five years respectively. The five-year put option costs more than the six-month option because the investor is protected for a longer period of time.
2. Second, the discount for incremental lack of marketability was computed. The six-month put option DLOM (16.2%) was subtracted from the five-year DLOM (26.6%) for an incremental lack of marketability of 10.40%. This incremental DLOM represents the discount for lack of marketability not incorporated within the partially liquid value that is based on the RELP pricing in the secondary market.
3. Third, the incremental DLOM (10.40%) was then divided by the DLOM from the five-year protective put option of 26.6%, which yielded 0.39, representing the proportion of DLOM not incorporated within the value that was based on RELP pricing in the secondary market ("Proportion").
4. Fourth, the Proportion (0.39) was multiplied by the complete DLOM developed from the FMV Study, Pre IPO Study, and protective put option of 30.0% to yield an incremental DLOM of (12.0%). This incremental DLOM was then applied to the value of the LP interest ($100.00) that included partial marketability. The product that includes the additional incremental DLOM ($88) is the nonmarketable, minority value of the private LP Interest.

Use of Protective Put Options to Determine Lack of Marketability. In a protective put option calculation, the cost of purchasing a put option to assure against downward price changes of the underlying asset is a proxy for the cost of marketability. This downside price protection represents a measurement for DLOM, because an investor who was given the choice between purchasing two identical securities would pay less for the nonmarketable security than for the partially marketable security. Factors that influence the value of downside price protection (DLOM) include:

1. The volatility of the underlying asset's price.
2. The holding period or term of the security.
3. The risk-free rate corresponding to the expected holding period.

Differences between Partial and Complete Lack of Marketability. Protective put option methodology can also be used to determine a DLOM for LP and LLC interests in closely held entities that own real estate, securities, or a combination of both. After the value of the LP or LLC interest is determined, and depending on the level of marketability of the comparable traded interests, a complete or incremental lack of marketability discount is applied. If the LP or LLC interest is valued using a completely marketable security, such as a REIT that can be exchanged for cash in three days, a complete DLOM developed from the FMV Study, Pre-IPO Study, and protective put option is applied without adjustment. If, however, the private LP or LLC interest is valued using a partially marketability security, such as a RELP, the complete DLOM must be reduced because the discount that is derived would reflect the complete lack of marketability discounts, whereas RELP pricing includes a partial lack of marketability discount. That is, applying a complete lack of marketability discount from the FMV and Pre-IPO Studies would overestimate the appropriate discount to the LP or LLC interest value that is based on RELP pricing.



4 Longstaff, "How Much Can Marketability Affect Security Value?," 50 J. Finance 1767 (December 1995).

5 Finnerty, "The Impact of Transfer Restrictions on Stock Prices," November 2007, available at http://www.fma.org/Prague/Papers/TheImpactofTransferRestrictions.pdf.

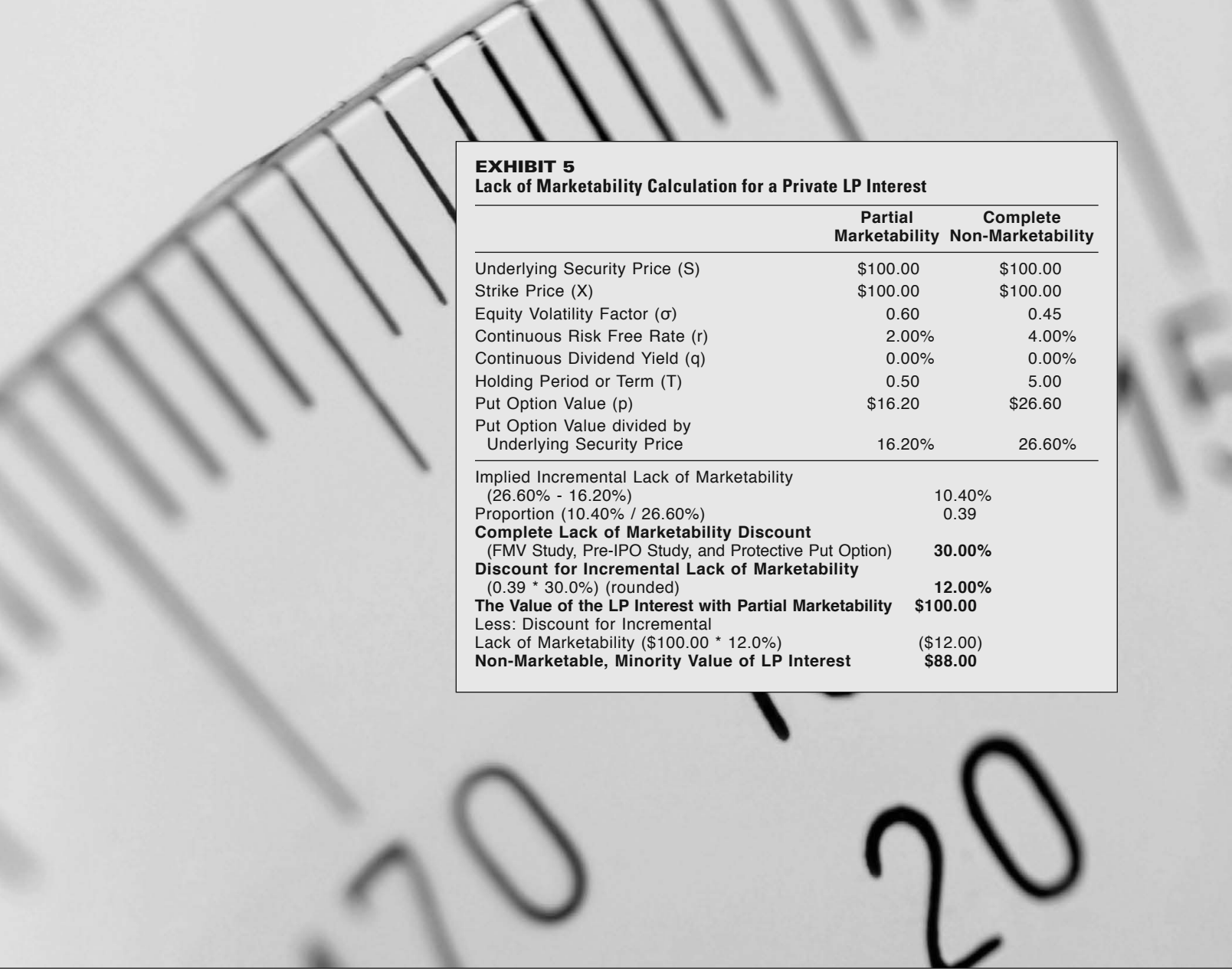


EXHIBIT 5
Lack of Marketability Calculation for a Private LP Interest

	Partial Marketability	Complete Non-Marketability
Underlying Security Price (S)	$100.00	$100.00
Strike Price (X)	$100.00	$100.00
Equity Volatility Factor (σ)	0.60	0.45
Continuous Risk Free Rate (r)	2.00%	4.00%
Continuous Dividend Yield (q)	0.00%	0.00%
Holding Period or Term (T)	0.50	5.00
Put Option Value (p)	$16.20	$26.60
Put Option Value divided by Underlying Security Price	16.20%	26.60%

Implied Incremental Lack of Marketability (26.60% - 16.20%)	10.40%	
Proportion (10.40% / 26.60%)	0.39	
Complete Lack of Marketability Discount (FMV Study, Pre-IPO Study, and Protective Put Option)	**30.00%**	
Discount for Incremental Lack of Marketability (0.39 * 30.0%) (rounded)	**12.00%**	
The Value of the LP Interest with Partial Marketability	**$100.00**	
Less: Discount for Incremental Lack of Marketability ($100.00 * 12.0%)	($12.00)	
Non-Marketable, Minority Value of LP Interest	**$88.00**	

Instead of applying the complete discount, the applicable discount is computed that captures the incremental lack of marketability between a partially marketable RELP interest and a completely nonmarketable private LP or LLC interest. This computation adjusts for the difference in marketability between a security with partial marketability (six months to sale) and a completely nonmarketable security (five-year holding period) using put options calculated with corresponding six-month and five-year terms. The proportionate difference between these put options of varied terms represents the incremental discount for lack of marketability not included within the minority value of the private LP or LLC interest.

Using put options to quantify the incremental DLOM that is not included within the minority value of the private LP or LLC interest is an approach that does not rely on empirical data from historical transactions, but on quantifiable parameters that are specific to the asset being valued. In addition, this incremental methodology corrects the potential for overestimation of the discount for lack of marketability when valuing an LP or LLC interest using partially liquid RELPs.

Alternative Methods for Completely Nonmarketable Securities

Earlier in this article, the BSM option pricing methodology was applied to determine the lack of marketability discount on a security that was assumed to be completely nonmarketable. While BSM is considered an appropriate methodology to determine this discount, two other options-based models exist to determine the discount:

1. A lookback put option model developed by Francis A. Longstaff (the "Longstaff model").[4]
2. An average strike put option model developed by John D. Finnerty (the "ASP model").[5]

The Longstaff Model. The Longstaff model considers a hypothetical investor with perfect market-timing ability who is restricted from selling a security for a fixed period. The value of marketability is the present value of the incremental cash flow an investor would receive if she were to swap the value of the security at the end of the restricted period for the maximum value of the security during the restricted period.

Unlike the BSM model, the Longstaff model captures the opportunity cost associated with the

EXHIBIT 6
The ASP Model

$$D(T) = \exp^{(r-q)T}\left[N\left(\frac{(r-q)T}{v\sqrt{T}}\right)+\frac{1}{2}\left(v\sqrt{T}\right)\right]-N\left(\frac{(r-q)T}{v\sqrt{T}}-\frac{1}{2}\left(v\sqrt{T}\right)\right)$$

$$v\sqrt{T} = \sqrt{\left[\sigma^2 T + \ln\left[2\left(\exp^{T\sigma^2}-\sigma^2 T-1\right)\right]\right]-2\ln\left(\exp^{T\sigma^2}-1\right)}$$

exp = a mathematical constant; the base of the natural logarithm
q = annualized dividend yield of security
r = risk-free rate
T = time to expiration of option (in years)
N = cumulative probability function for a standardized normal distribution
σ = annualized volatility of the underlying stock
D(T) = percentage discount for lack of marketability on the security

EXHIBIT 7
Comparison of Lack of Marketability Indications

	BSM	ASP Model
Price of the underlying asset/a/	$100.00	$100.00
Strike price	$100.00	N/A /b/
Annualized volatility of the underlying asset	0.45	0.45
Option term (in years)	5.00	5.00
Risk-free rate	4.00%	4.00%
Dividend yield	0.00%	0.00%
Resulting D(T)	$26.60	$35.93
Marketable, Minority Value of Security	$100.00	$100.00
Less: Discount for Lack of Marketability	($26.60)	($35.93)
Non-Marketable, Minority Value of Security	**$73.40**	**$64.07**

/a/ Marketable minority value of the LP or LLC interest, not net asset value.

/b/ The ASP Model is an average strike price model. Therefore, no explicit strike price is assumed in the model.

EXHIBIT 8
Lack of Marketability Calculation for Private LP Interest Using the ASP Model

	Partial Marketability	Complete Non-marketability
Underlying Security Price (S)	$100.00	$100.00
Equity Volatility Factor (σ)	0.60	0.45
Risk Free Rate (r)	2.00%	4.00%
Dividend Yield (q)	0.00%	0.00%
Holding Period or Term (T)	0.50	5.00
Put Option Value (p)	$10.16	$35.93
Put Option Value divided by Underlying Security Price	10.16%	35.93%

Implied Incremental Lack of Marketability (35.93% - 10.16%)	25.77%
Proportion (10.16% / 35.93%)	0.28
Complete Lack of Marketability Discount (FMV Study, Pre-IPO Study, and Protective Put Option)	30.00%
Discount for Incremental Lack of Marketability (0.28 * 30.0%)	8.40%
The Value of the LP Interest with Partial Marketability	$100.00
Less: Discount for Incremental Lack of Marketability ($100.00 * 12.0%)	($8.40)
Nonmarketable, Minority Value of LP Interest	$91.60

investor's ability to sell her security at the maximum attainable price during the restricted period rather than sell her security at the price at the end of the restricted period. The Longstaff model is a "lookback model," in that the investor is assumed to have perfect hindsight to observe the maximum price of the security during the restricted period and to sell the security at the precise moment the security reaches its maximum price in the market.

The Longstaff model is valuable in that it yields an upper-bound for the value of marketability that can be

compared to DLOM indications derived from other models or studies. Arguably, all DLOM indications derived from other models should be lower than the indication derived from the Longstaff model. But since investors do not have the ability to time the market perfectly, the Longstaff model does not produce meaningful point estimates of DLOM that can be applied to nonmarketable securities. Therefore, while it is important to consider the opportunity cost of the value foregone by holding an investment until option maturity rather than selling at some point within the option term, the Longstaff model will likely lead to an overestimation of the DLOM if applied directly to a security.

The ASP Model. Another DLOM model that captures opportunity cost is the ASP model. The ASP model derives a DLOM based on the value of an average strike put option, a specific type of Asian option whose strike price depends on the arithmetic average of the security's price during the life of the option. Unlike the Longstaff model, under the ASP model the investor is not assumed to have any special market timing ability. Rather, the investor is assumed to be equally likely to sell his security at any time during the assumed holding period. Consequently, the strike price of the put option in the ASP model is the arithmetic average of the forward stock prices during the life of the option.

Comparatively, the strike price of the lookback put option in the Longstaff model is the maximum forward price during the life of the option. In practice, therefore, the ASP model is a promising alternative to the BSM model, as it incorporates the opportunity cost of value foregone by holding an investment until option maturity rather than selling at some point within the option term, but it does not assume that the investor has any ability to time the sale of the security at the highest price. The ASP model is shown in Exhibit 6.

The BSM and the ASP Models Compared. As in the example presented earlier, consider a hypothetical investor who owns a security with a marketable, minority value of $100 at the valuation date with a holding period of five years. Also, assume an annualized volatility for the security of 0.45, a risk-free rate of 4.0%, and a dividend yield of 0.0%. Exhibit 7 provides a comparison of DLOM indications using the BSM model versus the ASP model for a security that is completely nonmarketable.

In Exhibit 7, the ASP model yields a discount (35.93%) that is 933 basis points higher than the discount from the BSM (26.60%). The 933-basis-point difference can be viewed as a measure of the opportunity cost associated with the investor's inability to sell the security during the restricted period. It reflects value that is foregone assuming that the investor must hold on to the security until option expiration rather than sell the security with equal probability at any given time within the option term. Like the BSM model, the ASP Model can be applied to securities that have partial marketability. Exhibit 8 presents the application of the ASP model to securities that have partial marketability.[6]

Conclusion

Valuation of a protective put option using the BSM model is a method used to determine a lack of marketability discount for a nonmarketable security (e.g., a closely held common stock, minority LP interest, or minority LLC interest). Although quantitative models, such as the BSM model, yield discounts for fully marketable securities, the application of protective put theory can be extended to measure the partial discounts for lack of marketability on RELP interests in comparison to closely held minority securities.

The Longstaff and the ASP models serve as alternatives to the BSM model. The primary advantage of the Longstaff and ASP models is their ability to capture the opportunity cost of the value foregone by holding an investment until option maturity, rather than selling at some point within the option term. One drawback of the Longstaff model, however, is that it assumes the investor has perfect market timing ability, and that the sale of the security will be at the highest possible price during the option term. The ASP model resolves this issue, as it incorporates the opportunity cost of value foregone by holding an investment until option maturity rather than selling at some point within the option term, but does not assume that the investor has any type of special market timing ability. This feature of the ASP model makes it a promising alternative to the BSM model. ●

6 See Exhibit 5 and the accompanying text for a full discussion regarding the determination of DLOM for partially marketable securities.

601 California Street, Suite 800
San Francisco, CA 94108
415.392.0888 voice
415.392.7070 fax